

Checkpoints Chapter 5 Projectiles**Question 163**

The vertical and horizontal components are equal, so the angle must be 45°

Question 164

Use Pythagoras,

$$v^2 = 13.8^2 + 18.4^2$$

$$\therefore v^2 = 529$$

$$\therefore v = 23 \text{ m/s} \quad (\text{ANS})$$

Question 165

The small part will fly off tangentially. It will go straight up therefore it will land directly below A, at close to S.

$$\therefore S \quad (\text{ANS})$$

Question 166

$$E_{\text{tot}} = \text{constant} = KE_{\text{bottom}} = KE_{\text{top}} + PE_{\text{top}}$$

$$E_{\text{tot}} = 1860 = 660 + mgh$$

$$\therefore 1860 = 660 + 60 \times 10 \times h$$

$$\therefore h = \frac{1860 - 660}{60 \times 10}$$

$$\therefore h = 2.0 \text{ m} \quad (\text{ANS})$$

Question 167

Using $KE_{\text{top}} = 660$

$$= \frac{1}{2} \times m \times v^2$$

$$= \frac{1}{2} \times 60 \times v^2$$

$$\therefore v = 4.7 \text{ m/s} \quad (\text{ANS})$$

Question 168

The distance is 1.5m, (from the diagram) you must take the displacement to be between the start point and the final resting place, \therefore vertically down.

$$\therefore 1.5 \text{ m, vertically down} \quad (\text{ANS})$$

Question 169

The time taken will be the time to travel to the top of the flight and then down to the ground. Considering the upward motion, take up to be positive.

$v = u + at$ becomes $v = u - gt$ the acceleration due to gravity is down.

$$\therefore 0 = 6.5 \sin 30 - 10 \times t$$

$$\therefore t = 0.325 \text{ s to go up}$$

We must calculate the maximum height of the ball. This is given by

$$v^2 = u^2 + 2ax$$

$$\therefore 0^2 = (6.5 \sin 30)^2 - 2 \times 10 \times x$$

$$\therefore x = 0.528 \text{ m}$$

The total height of the ball above the ground is given by $1.5 + 0.528 = 2.028 \text{ m}$

Considering the downward motion, take down to be positive.

$$x = ut + \frac{1}{2} at^2$$

$$\therefore 2.028 = 0 + \frac{1}{2} \times 10 \times t^2$$

$$\therefore t = 0.637 \text{ s to come down}$$

the total travel time is the time to go up plus the time to go down which equals

$$0.325 + 0.637 = 0.96 \text{ s} \quad (\text{ANS})$$

Question 170

The horizontal velocity is given by

$$v \cos 30 = 5.63 \text{ m/s}$$

\therefore the horizontal distance travelled (the range) equals

$$5.63 \times 0.96$$

$$= 5.4 \text{ m} \quad (\text{ANS})$$

Question 171

The time the ball takes to fall 45 m is the same time as it takes to travel the 155 m.

$$\text{Use } x = ut + \frac{1}{2} at^2$$

$$\therefore 45 = 0 + \frac{1}{2} \times 10 \times t^2$$

$$\therefore t^2 = 9$$

$$\therefore t = 3 \text{ secs}$$

In this time it travels 155 m.

$$\therefore v = \frac{d}{t}$$

$$= \frac{155}{3}$$

$$= 52 \text{ m/s} \quad (\text{ANS})$$

Question 172

The initial angle of elevation is 30° .

$$\begin{aligned} \text{Then } v_{\text{vertical}} &= v_0 \sin 30^\circ \\ &= 50 \times 0.5 \\ &= 25 \text{ m/s} \end{aligned}$$

$$\text{Use } v^2 = u^2 - 2gh$$

$$\therefore \text{At the top } v_{\text{vertical}} = 0,$$

$$\therefore 0 = 25^2 - 2 \times 10 \times h$$

$$\therefore h = 625 \div 20$$

$$\therefore h = 31.25 = 31 \text{ m} \quad (\text{ANS})$$

Question 173

$$\begin{aligned} \text{Use } v &= u - gt \\ \therefore 0 &= 25 - 10 \times t \\ \therefore t &= 2.5 \end{aligned}$$

This is how long it takes to reach its maximum height. It then needs to drop a total of $31.25 + 45 = 76.25\text{m}$

$$\begin{aligned} \text{Use } h &= ut + \frac{1}{2}at^2 \\ \therefore 76.25 &= 0 + \frac{1}{2} \times 10 \times t^2 \\ \therefore t^2 &= 15. \\ \therefore t &= 3.905 \text{ secs} \\ \therefore \text{total time} &= 3.905 + 2.5 \\ &= 6.405 \\ &= \mathbf{6.4 \text{ sec}} \quad \text{(ANS)} \end{aligned}$$

Question 174

If the time of flight was 6.405 (**don't round off**) sec,

$$\begin{aligned} \text{then the range} &= v_{\text{horizontal}} \times \text{time} \\ &= 50\cos 30^\circ \times 6.405 \\ &= 277.34 \\ &= \mathbf{277 \text{ m}} \quad \text{(ANS)} \end{aligned}$$

Question 175

This is a question that is best solved using the range formula. I think that you should have this formula on your cheat sheet, but be VERY careful when using it.

$$\begin{aligned} \therefore R &= \frac{v^2 \sin 2\theta}{g} \\ \therefore 100 &= \frac{v^2 \times \sin 60}{10} \\ \therefore v^2 &= 1000 \div \sin 60^\circ \\ \therefore v^2 &= 1154.7 \\ \therefore v &= \mathbf{34 \text{ m/s}} \quad \text{(ANS)} \end{aligned}$$

Question 176

This is another question that is best solved using the range formula.

$$\begin{aligned} \therefore R \frac{v^2 \sin 2\theta}{g} &= \frac{v^2 \sin 2\theta}{g} \\ \therefore 100 &= \frac{v^2 \times \sin 90}{10} \\ \therefore v^2 &= 1000 \div \sin 90^\circ \\ \therefore v^2 &= 1000 \\ \therefore v &= \mathbf{31.6 \text{ m/s}} \quad \text{(ANS)} \end{aligned}$$

Question 177

$$\begin{aligned} \text{The vertical displacement} &= 3.5 - 2.1 \\ &= \mathbf{1.4 \text{ m}} \quad \text{(ANS)} \end{aligned}$$

Question 178

The total displacement is the difference between the final and initial positions.

Vertically the difference is 1.4m

Horizontally the difference is 3.4m

Use Pythagoras to find the displacement.

$$\begin{aligned} x^2 &= 1.4^2 + 3.4^2 \\ \therefore x^2 &= 13.52 \\ \therefore x &= 3.677 \\ \therefore x &= \mathbf{3.7\text{m}} \quad \text{(ANS)} \end{aligned}$$

Question 179

In the horizontal direction, the ball travels 3.4 m in 1.1 secs

$$\begin{aligned} \therefore v &= \frac{d}{t} \\ &= \frac{3.4}{1.1} \\ &= 3.09 \\ \therefore v &= \mathbf{3.1\text{m/s}} \quad \text{(ANS)} \end{aligned}$$

Question 180

In the vertical direction the displacement

$$\begin{aligned} y &= ut - \frac{1}{2}gt^2 \\ 1.4 &= u \times 1.1 - \frac{1}{2} \times 10 \times 1.1^2 \\ 1.4 + 6.05 &= u \times 1.1 \\ \therefore u &= 6.77 \\ \therefore u &= \mathbf{6.8 \text{ m/s}} \quad \text{(ANS)} \end{aligned}$$

Question 181

Use Pythagoras to find the launch velocity. (Don't use your rounded off figures)

$$\begin{aligned} v^2 &= 3.09^2 + 6.77^2 \\ \therefore v^2 &= 55.381 \\ \therefore v &= 7.44 \\ \therefore v &= \mathbf{7.4 \text{ m/s}} \quad \text{(ANS)} \end{aligned}$$

Question 182

$$\begin{aligned} \text{Use } \tan \theta &= \frac{V_{\text{vertical}}}{V_{\text{horizontal}}} \\ &= \frac{6.77}{3.09} \\ \therefore \theta &= 65.47^\circ \\ &= \mathbf{65^\circ} \quad \text{(ANS)} \end{aligned}$$

Question 183

Initially the car is moving horizontally, so the initial vertical velocity must be zero

Question 184

In the vertical direction the velocity

$$v = u + gt$$

$$v = 0 + 10 \times 1.8$$

$$v = 18 \text{ m/s} \quad (\text{ANS})$$

Question 185

The sum of the forces in the horizontal direction = 0. \therefore the air resistance must be very small. The force of the car's engine, cannot 'balance' the effects of air resistance, because there is nothing for the wheels to have friction against, to create a force to the right.

$$\therefore \text{B} \quad (\text{ANS})$$

Question 186

Use $x = ut + \frac{1}{2}at^2$, where $u = 0$.

$$\therefore x = \frac{1}{2} \times 10 \times 1.8^2$$

$$\therefore x = 16.2 \text{ m} \quad (\text{ANS})$$

Question 187

The horizontal velocity = 10 m/s. The vertical velocity is 18 m/s.

Use Pythagoras to add these two vectors, to find the sum.

$$v^2 = 18^2 + 10^2$$

$$\therefore v^2 = 424$$

$$\therefore v = 20.6 \text{ m/s} \quad (\text{ANS})$$

Question 188

A C E (ANS)

Question 189

$$\text{KE} = \frac{1}{2}mv^2$$

$$\therefore 110 = \frac{1}{2} \times 0.550 \times v^2$$

(Remember to convert the mass to kilograms)

$$\therefore v^2 = \frac{2 \times 110}{0.550}$$

$$\therefore v = 20 \text{ ms}^{-1} \quad (\text{ANS})$$

Question 190

At the ground level $\text{TE} = \text{KE} + \text{PE}$, ($\text{PE} = 0$)

$$\therefore \text{TE} = 110\text{J}$$

At the top $\text{TE} = 110\text{J}$

$$= \text{KE} + \text{PE}$$

$$\therefore 110 = 0.55 \times 10 \times 8 + \text{KE}$$

$$\therefore \text{KE} = 110 - 44$$

$$\therefore \text{KE}_{\text{top}} = 66 \text{ J} \quad (\text{ANS})$$

Question 191

$$\therefore \frac{1}{2}mv^2 = 66$$

$$\therefore v^2 = \frac{2 \times 66}{0.55}$$

$$\therefore v^2 = 240$$

$$\therefore v = 15.49$$

$$\therefore v = 15.5 \text{ ms}^{-1} \quad (\text{ANS})$$

Question 192

The height that the projectile reaches is given by

$$v^2 = u^2 - 2gh$$

$$\therefore 0^2 = (u \sin \theta)^2 - 2 \times 10 \times 8$$

$$\therefore 0 = (20 \sin \theta)^2 - 160$$

$$\therefore \sin^2 \theta = 160 \div 400$$

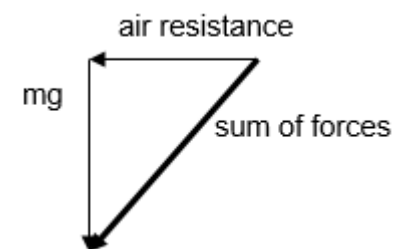
$$\therefore \sin \theta = \sqrt{0.4}$$

$$\therefore \theta = 39.23^\circ$$

$$\therefore \theta = 39^\circ \quad (\text{ANS})$$

Question 193

The resultant force is the sum of the two forces acting. The two forces are the air resistance that is opposing the motion, and the weight force.



$$\therefore F \quad (\text{ANS})$$

Question 194

Newton's first law will explain the motion, and that is a body in motion will continue in motion unless a net force acts on it. Initially the parcel was travelling at the same speed as the car. When Fred braked to avoid the collision, the parcel continued in its motion as the very small frictional force (between seat and parcel) was not sufficient to stop the motion.

Question 195 (2010 Q10, 2m, 65%)

The time it will take for the package to land is given by $x = ut + \frac{1}{2}at^2$

$$\therefore 200 = 0 + \frac{1}{2} \times 10 \times t^2$$

$$\therefore t^2 = 40$$

$$\therefore t = 6.32 \text{ s}$$

The horizontal velocity will be unchanged, as we are ignoring air resistance,

$$\therefore d = u \times t$$

$$\therefore d = 63.2 \text{ m (ANS)}$$

Question 196 (2010 Q11, 2m, 55%)

The speed will have both a horizontal component of 10 m/s as well as a vertical component. The vertical component comes from $v = u + at$,

$$\begin{aligned} \therefore v &= 0 + 10 \times 6.32 \\ &= 63.2 \text{ m/s} \end{aligned}$$

Using Pythagoras we get speed

$$\begin{aligned} &= \sqrt{10^2 + 63.2^2} \\ &= 64 \text{ m/s (ANS)} \end{aligned}$$

(If we consider this in km/hr, it becomes 230 km/hr. You wouldn't really want to be in the way of the parcel).

Question 197

This question was not on the exam.

We are instructed to ignore air resistance. Therefore there will not be any energy losses. This means that the final speed will be exactly the same as the initial speed, as they are both at the same height.

$$\therefore 24 \text{ m/s (ANS)}$$

Question 198 (2011 Q12, 2m, 70%)

Consider the vertical motion

Use $v = u - gt$

$$\text{where } 0 = 24\sin 37^\circ - 10t$$

$$\therefore t = 1.4444$$

This is the time it takes to get to the top (where the vertical velocity = 0). Therefore it will take twice as long to reach B.

$$\therefore t = 2.89 \text{ sec (ANS)}$$

Question 199 (2011 Q13, 2m, 65%)

Use $v^2 - u^2 = 2gx$

$$\therefore x = \frac{(24\sin 37^\circ)^2}{20}$$

$$\therefore x = 10.43 \text{ m (ANS)}$$

Question 200 (2012 Q6a, 3m, 63%)

Using the velocities in the vertical direction, and $v^2 - u^2 = 2gx$, for the motion on the way up to the top of the flight.

$$\therefore 0 - (u\sin 60^\circ)^2 = 2 \times -10 \times 15$$

$$\therefore -u^2 \times \left(\frac{\sqrt{3}}{2}\right)^2 = -300$$

$$\therefore u^2 = 400$$

$$\therefore u = 20 \text{ m/s (ANS)}$$

Question 201 (2012 Q6b, 2m, 65%)

The time it takes to get to the top of the flight will be half the time of flight.

Use $v = u - gt$ to get

$$0 = 20\sin 60^\circ - 10t$$

$$\therefore 0 = 17.32 - 10t$$

$$\therefore t = 1.7 \text{ sec}$$

$$\therefore \text{Total time} = 3.5 \text{ sec (ANS)}$$

Question 202 (2013 Q8a, 3m, 50%)

The methodical way to complete this is to divide the problem into two parts, up and down.

Consider 'up'

Initial velocity is 10 m/s

Final velocity = 0

$$\therefore v = u - gt$$

$$\therefore 0 = 10 - 10t$$

$$\therefore t = 1 \text{ sec.}$$

Height at top

$$x = ut - \frac{1}{2}gt^2$$

$$\text{gives } x = 10 \times 1 - \frac{1}{2} \times 10 \times 1^2$$

$$\therefore x = 5 \text{ m}$$

$$\therefore \text{height} = 20 \text{ m.}$$

Consider 'down'

$$x = ut + \frac{1}{2}gt^2$$

$$\therefore 20 = 0 + 5t^2$$

$$\therefore t^2 = 4$$

$$\therefore t = 2$$

$$\begin{aligned} \therefore \text{Total time} &= 1.0 + 2.0 \\ &= 3.0 \text{ sec (ANS)} \end{aligned}$$

Question 203 (2013 Q8b, 3m, 40%)

The horizontal component of the velocity will remain constant at

$$v_H = 20 \cos 30^\circ \\ = 17.32 \text{ m/s.}$$

The vertical component will be

$$v = u + gt$$

on the way down, $u = 0$, $g = 10$, $t = 2$

$$\therefore v = 20 \text{ m/s.}$$

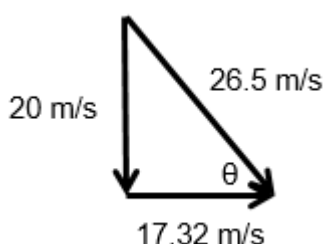
Use Pythagoras to find the magnitude of the velocity.

$$\therefore 20^2 + 17.32^2 = v^2$$

$$\therefore v^2 = 700$$

$$\therefore v = 26.5 \text{ m/s}$$

To find the angle use



$$\text{Use } \tan \theta = \frac{20}{17.32}$$

$$\therefore \theta = 49.1^\circ$$

$$\therefore v = 26.5 \text{ ms}^{-1} \text{ at an angle of } 49.1^\circ$$

$$\therefore \mathbf{26.5 \text{ m/s } \quad 49.1^\circ \text{ (ANS)}}$$

Question 204 (2014 Q3a, 2m, 75%)

Use the initial vertical component of the velocity.

$$v_{\text{vertical}} = 20 \times \sin 30^\circ \\ = 10$$

In the vertical direction, use

$$v^2 - u^2 = 2gx$$

$$\therefore 0^2 - 10^2 = 2 \times -10 \times x$$

$$\therefore 100 = 20 \times x$$

$$\therefore \mathbf{x = 5m \quad \text{(ANS)}}$$

Question 205 (2014 Q3b, 3m, 53%)

To find the time that it takes for the ball to hit the advertising board, you need to know how long it takes to travel the 26 m in the horizontal.

Use $d = v \times t$

$$\therefore 26 = 20 \cos 30^\circ \times t$$

$$\therefore t = 1.5$$

In the vertical.

Then use $s = ut - \frac{1}{2} \times 10 \times 1.5^2$

$$\therefore s = 20 \sin 30^\circ \times 1.5 - 11.26$$

$$\therefore s = 10 \times 1.5 - 11.26$$

$$\therefore s = 3.73 \text{ m}$$

$$\therefore \mathbf{3.7 \text{ m (ANS)}}$$