Gravity

Newton's law of gravitation

The force of attraction between any two objects:

- depends on the mass of the two objects. (More mass = more attraction)
- depends on the square of the separation. (Further apart = less attraction)
- is never zero.

G is a constant: $G = 6.67 \times 10^{-11} Nm^2/kg^2$

$$F = \frac{-GMm}{r^2}$$

Masses of each object

Distance between centres of mass

Gravitational field strength

The gravitational field strength determines the acceleration of falling objects.

$$F = \frac{-GMm}{r^2}$$

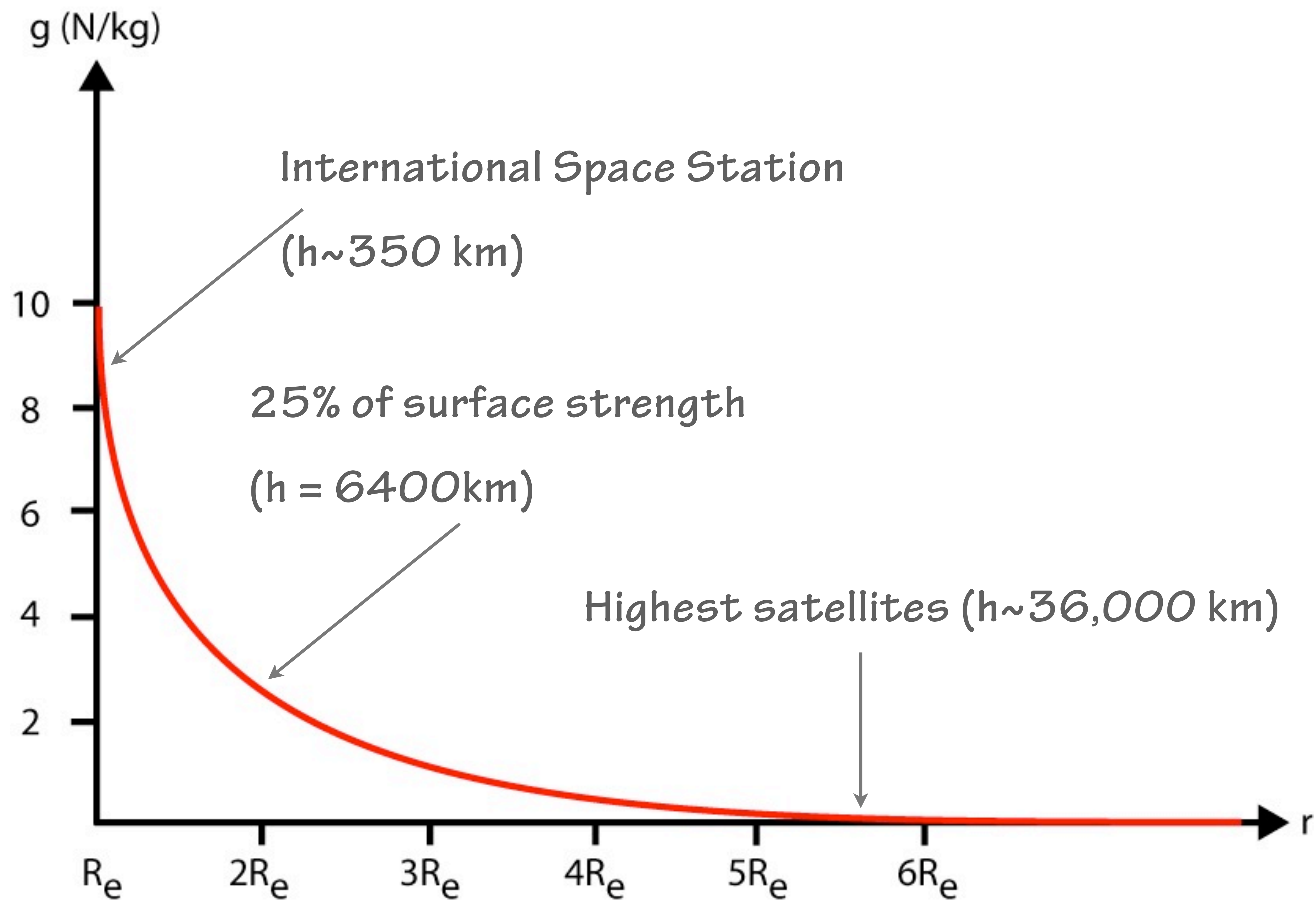
$$a = \frac{F}{m} = \frac{GM}{r^2}$$

Gravitational acceleration - just depends on the mass of the planet & distance from centre.

At the surface of the Earth:

$$a = \frac{(6.7\times10^{-11}Nm^2kg^{-2})(6.0\times10^{24}kg)}{(6.4\times10^6 m)^2} = 9.8m/s^2$$

Gravitational field strength



Work & Gravity

- Recall that work is the product of force and displacement.
- Work : $W = Fx$
- Gravitational potential energy: $E_p = mg\Delta h$
- This is an approximation at the surface where the strength of gravity is constant.
- As an object is lifted higher, less work is required due to the decreasing strength of gravity.
- At the height of the International Space Station, the gravitational field is still about 90% as strong as the surface.

Gravitational Potential Energy

- E_p can be evaluated by:
- Estimating the area under force (gravity) distance graph.
- Integration of the work function to find the exact area. (Not required for VCE study design.)

$$W = \Delta E_p = \int_{r_e}^{h+r_e} F(r)dr = \int_{r_e}^{h+r_e} \frac{GMm}{r^2} = -\left[\frac{GMm}{r}\right]_{r_e}^{h+r_e}$$

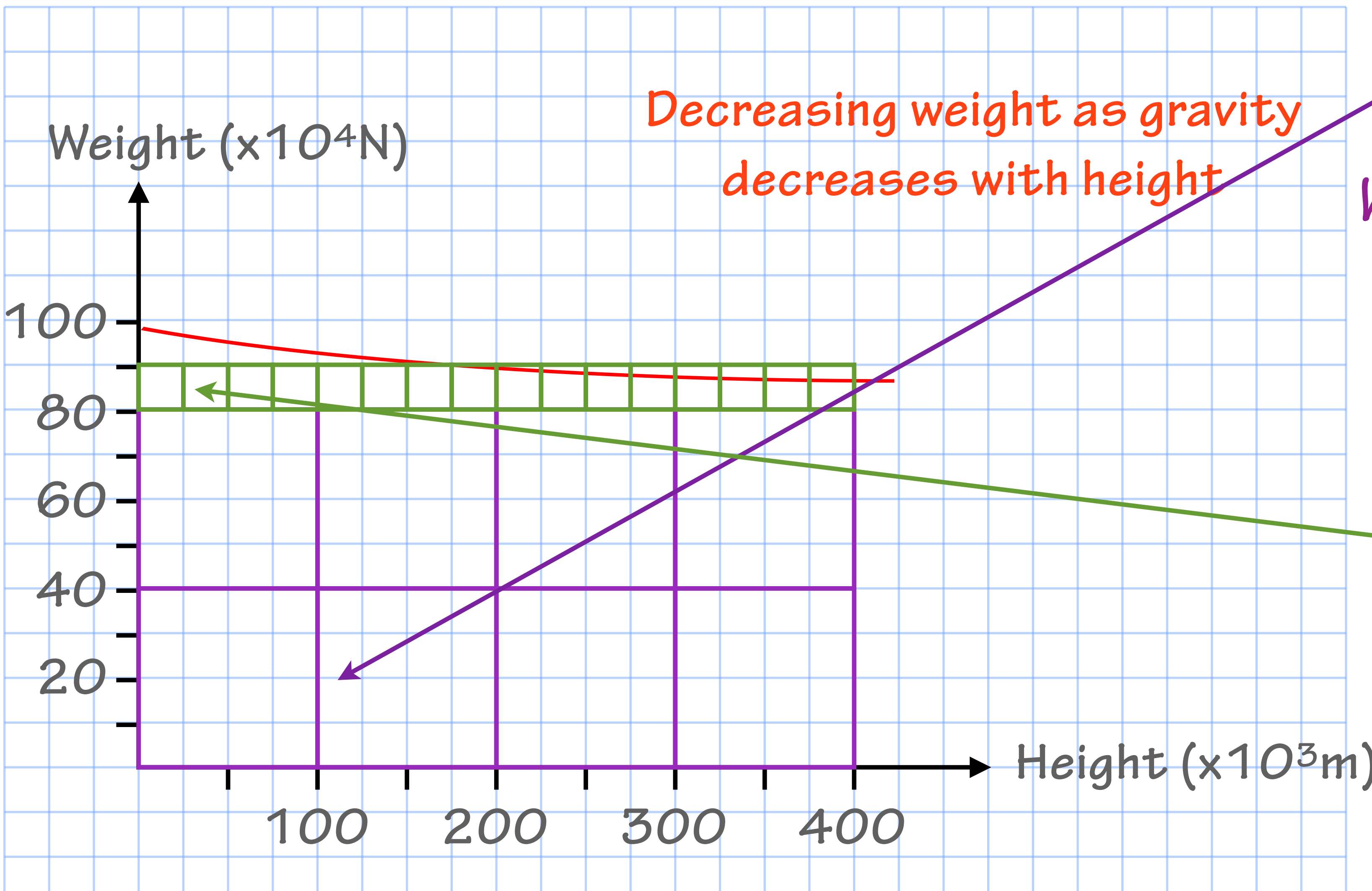
Starting height (ground level is not 0m r = 6.4×10^6 m)

$$\text{Absolute Gravitational Potential Energy} = -\frac{mGM}{r}$$

- All absolute E_p values are negative, but higher up are closer to 0.

Potential energy calculations - area under graph

Find the work done to lift the 100 ton space shuttle 400 km above the Earth.



Large squares:

$W = Fx$

$W = 40 \times 10^4 N \times 100 \times 10^3 m$

$W = 4 \times 10^{10} J$

$8 \times 4 \times 10^{10} J = 3.2 \times 10^{11} J$

Small squares:

$W = 10 \times 10^4 N \times 25 \times 10^3 m$

$W = 2.5 \times 10^9 J$

$16 \times 2.5 \times 10^9 J = 4.0 \times 10^{10} J$

Total work:

$W = 3.6 \times 10^{11} J$

Potential energy calculations - using work function

$$\text{Absolute Gravitational Potential Energy} = -\frac{mGM}{r}$$

At the surface:

$$E_p = -\frac{(1.00\times10^{5}\,kg)(6.67\times10^{-11}\,Nm^{2}kg^{-2})(5.97\times10^{24}\,kg)}{6.37\times10^{6}\,m}$$

$$E_p = -6.25\times10^{12}\,J$$

At 400km:

$$E_p = -\frac{(1.00\times10^{5}\,kg)(6.67\times10^{-11}\,Nm^{2}kg^{-2})(5.97\times10^{24}\,kg)}{6.77\times10^{6}\,m}$$

$$E_p = -5.88\times10^{12}\,J$$

$$W = \Delta E_p = 3.7\times10^{11}\,J$$

Escape velocity

- Given enough kinetic energy, a projectile will be able to escape the gravitational pull of the Earth.
- The minimum kinetic energy needed would be enough energy to overcome the gravitational potential energy requirements.

Kinetic energy $\frac{1}{2}mv^2 = \frac{GMm}{r}$ Absolute gravitational energy

$$v = \sqrt{\frac{2GM}{r}}$$

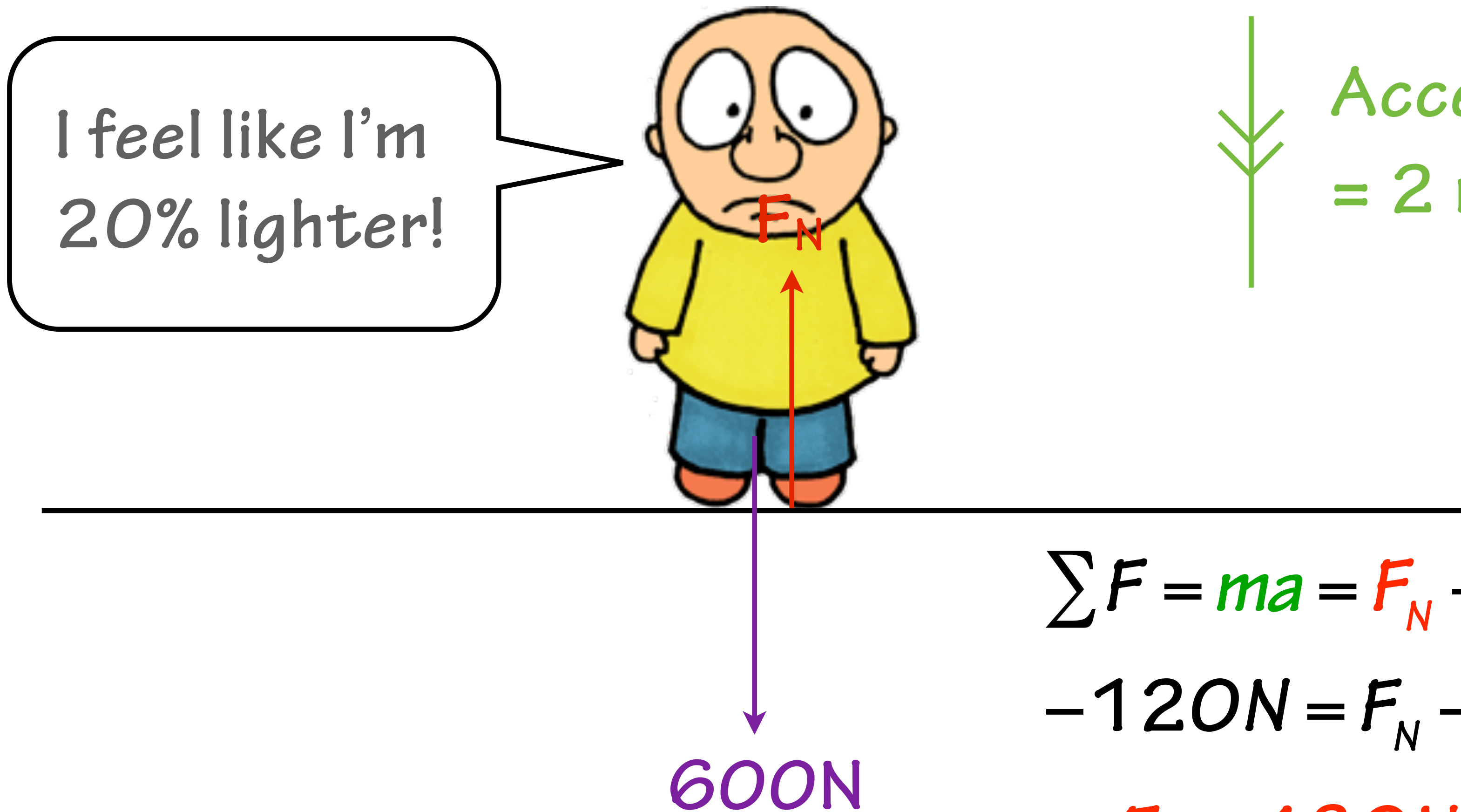
$$v = \sqrt{\frac{2(6.7\times10^{-11} Nm^2/kg^2)(6.0\times10^{24} kg)}{6.4\times10^{6} m}}$$

$$v \approx 11 km/s$$

Apparent weight

- THERE IS NO FEELING OF WEIGHT IN ORBIT…….. BUT NOT BECAUSE THERE IS NO GRAVITY!
- If there was no gravity - how would the space shuttle still be in orbit?
- What we feel as weight is the force of the surface pushing up on us (Normal reaction force).
- In orbit, the satellite is falling down as it moves forward.
- How do you feel in a lift with a downwards acceleration?

A 60kg person accelerating down at 2 m/s².



$$\sum F = ma = F_N - mg$$

$$-120N = F_N - 600N$$

$$F_N = 480N$$

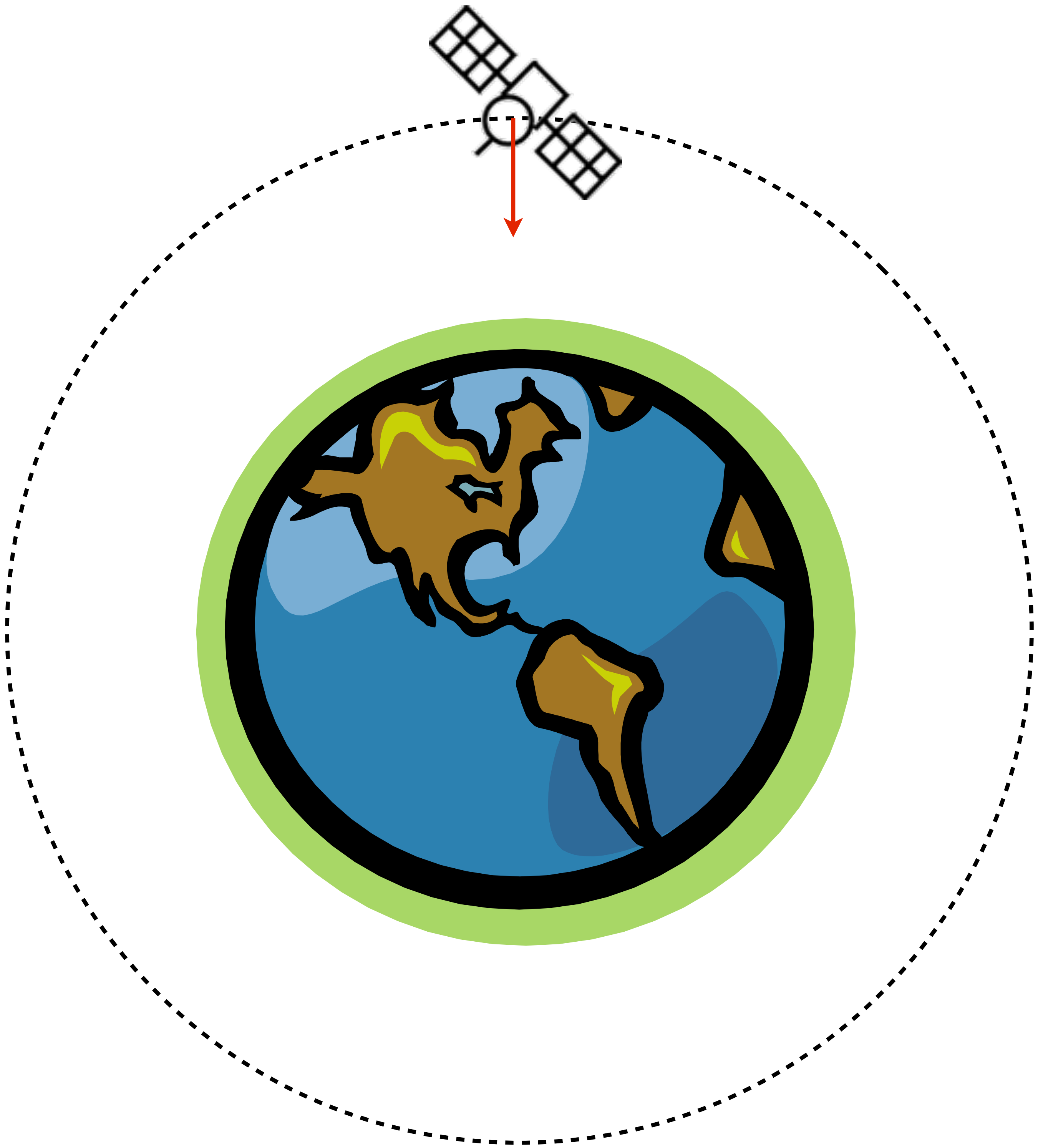
If the floor of the lift was to be accelerating at the rate of gravity:

$a = g \therefore mg = ma \therefore F_N = 0$

This is the situation in an orbiting satellite: there is no upwards force acting on the occupants.

Apparent weight

Acceleration down towards the Earth

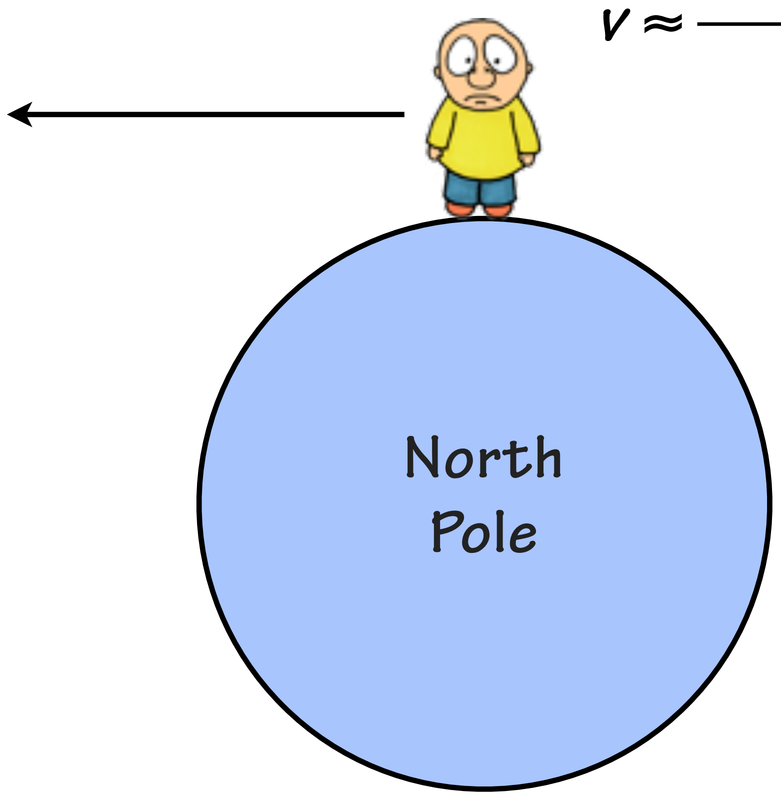


Gravity on the Earth

Objects weigh less at the equator for two reasons:

- The Earth is wider at the equator due to spinning while molten. Gravity is less as objects are further from the centre of the Earth (varies with $1/r^2$).
- The surface is "falling away" - accelerating towards the centre of the Earth.

$$v \approx \frac{40{,}000km}{24h} \approx 1700km/h$$



The view of the rotating Earth from the North Pole

$$a = \frac{4\pi^2 R}{T^2} = 0.03m/s^2$$

Objects will weigh less due to the spin of the Earth - by up to 0.3%.

Gravity - varies with latitude