## The Spring pendulum

Point $B$


## The Spring pendulum

- A mass of 1.0 kg is attached to a long spring of stiffness $\mathrm{k}=30$ $\mathrm{N} / \mathrm{m}$. The mass is allowed to fall from the starting height, at which the spring was unstretched.
- Calculate the distance $(x)$ over which the mass will oscillate.
- Calculate the maximum speed reached.
- Calculate the \% of each energy type at the three points.


## Conservation of energy: maximum extension

Kinetic energy $\longrightarrow E_{k}=m g x-\frac{1}{2} k x^{2}$
Difference of gravitational \& elastic energy

At point $c$, all of the potential energy from point a has been converted into elastic energy. $E_{k}=O$ at both points.

$$
O=m g x-\frac{1}{2} k x^{2} \quad O=x\left(m g-\frac{1}{2} k x\right) \longrightarrow x=\frac{2 m g}{k}=0.67 m
$$

## Conservation of energy: maximum speed

$$
\begin{array}{ll}
E(x)=m g x-\frac{1}{2} k x^{2} & \text { Kinetic energy as a function of height } \\
E^{\prime}(x)=m g-k x & \text { Derivative of kinetic energy }
\end{array}
$$

Maximum kinetic energy occurs at: $O=m g-k x \quad x=\frac{m g}{k}$

$$
x=0.33 m
$$

(That's where there is zero net force - it slows down as it gets past this.)
Potential energy $=\mathrm{mgx}=3.3 \mathrm{~J}$
Elastic energy $=1 / 2 k x^{2}=1.7 \mathrm{~J}$ Kinetic energy $=1.7 \mathrm{~J}$

$$
v=\sqrt{\frac{2 E_{k}}{m}}
$$

$$
v=1.8 \mathrm{~m} / \mathrm{s}
$$

## Conservation of energy: energy balance

| Energy form | $a$ (highest) | $b$ (mid point) | $c$ (lowest) |
| :---: | :---: | :---: | :---: |
| Potential | $6.7 \mathrm{~J}=100 \%$ | $3.3 \mathrm{~J}=50 \%$ | $O \mathrm{~J}=0 \%$ |
| Kinetic | $O \mathrm{~J}=0 \%$ | $1.7 \mathrm{~J}=25 \%$ | $O \mathrm{~J}=0 \%$ |
| Elastic | $O \mathrm{~J}=0 \%$ | $1.7 \mathrm{~J}=25 \%$ | $6.7 \mathrm{~J}=100 \%$ |

## Energy vs extension



Energy of spring pendulum

## maximum gravitational potential Minimum height: Maximum extension:

-Gravitational potential energy
Elastic potential energy
Kinetic Energy

## Energy vs time



