Checkpoints Chapter 7 GRAVITY

Question 269

To do this question you must get an equation that has both R and T, where R is the orbital radius and T is the period.
You can use Kepler's Law, which is;

$\frac{R^3}{T^2} = \text{constant}$. This is a very useful formula, even though it is not strictly on the course.

Therefore any increase in R must also have an increase in T.
Kepler's Law shows there is no dependence on mass.

Question 270

$$\frac{R^3}{T^2} = \text{constant}$$

$$\frac{(1.1\times10^9)^3}{7.16^2} = \frac{(1.87\times10^9)^3}{T^2}$$

(you could use any other set of data)
Solution of the equation yields

T = 17 days (ANS)

Question 271

Use $g = \frac{4\pi^2 R}{T^2}$

Where $R = \frac{C}{2\pi}$

$= \frac{2.29\times10^9}{2\pi}$

(don't forget to convert from km to m)

$= 3.64 \times 10^8$

and $T = 27.3 \times 24 \times 3600$

$= 2.36 \times 10^6$

$$\therefore g = \frac{4\pi^2 \times 3.64\times10^8}{(2.36\times10^6)^2}$$

$= 0.0026\ \text{ms}^{-2}$ (ANS)

Question 272

$$F = \frac{GMm}{R^2}$$

$$\therefore F = \frac{6.67\times10^{-11}\times1\times1}{(0.4)^2}$$

$\therefore F = 4.17\times10^{-10}$N (ANS)

Question 273 (This is not on the course)

A satellite orbits about the centre of the Earth. If it is not in the plane of the equator it will be alternately above a point in the Northern then Southern hemisphere. To remain above a fixed point on the Earth it must be above the equator.

Question 274

Gravitational Potential energy is given by

$$U = -\frac{GMm}{R} \Rightarrow U = -\frac{6.67\text{x}10^{-11}\times6.0\text{x}10^{24}\times2.0\text{x}10^5}{6.6\text{x}10^6}$$

$\therefore U = -12.13\text{x}10^{12}$ J

Note that the radius is the radius of the earth plus 200 km.
The kinetic energy is half the size of the Potential energy when in stable circular orbit so
$E_k = 6.1 \times 10^{12}$ J

The other way of doing this is:

$$\frac{mv^2}{r} = \frac{GMm}{R^2}$$

$$\therefore mv^2 = \frac{GMm}{R}$$

$$\therefore \tfrac{1}{2}mv^2 = \tfrac{1}{2}\times\frac{GMm}{R}$$

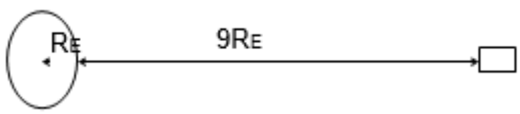
$$\therefore KE = \tfrac{1}{2}\times 12.13\text{x}10^{12}$$

$E_k = 6.1 \times 10^{12}$ J (ANS)

Question 275

Period is independent of mass, merely dependent on radius. The answer is 1

Question 276



Therefore the shuttle is 10 R_E from the centre of the Earth.
g at the Earth's surface is 9.8ms^{-2}

$g = \frac{GM}{R^2}$

We will use g_1 as the gravitational field at the surface of the Earth.

We will use g_2 as the gravitational field at $10\ R_E$ from the centre of the Earth.

$$\frac{g_2}{g_1} = \frac{GM}{R_2^2} \times \frac{R_1^2}{GM}$$

$$\frac{g_2}{g_1} = \frac{R_1^2}{R_2^2}$$ Sub values in for R_1 and R_2

$$\frac{g_2}{g_1} = \left(\frac{1R_E}{10R_E}\right)^2 \quad \frac{g_2}{g_1} = \left(\frac{1}{10}\right)^2 \quad \frac{g_2}{g_1} = \frac{1}{100}$$

get g_2 by itself

$$g_2 = \frac{g_1}{100}$$

$\therefore g_2 = 0.098\ ms^{-2}$

Using the equation of motion:

$v = u + at$

$\therefore v = 0 + 0.098 \times 200$

$\therefore v =$ **$20\ ms^{-1}$ (ANS)**

Question 277

Equating gravitational field strengths,

$$\frac{GM_S}{(R-x)^2} = \frac{GM_E}{x^2}$$

We can simplify such that $\frac{M_S}{(R-x)^2} = \frac{M_E}{x^2}$

Question 278

Use

$$W = mg = \frac{mv^2}{r} = \frac{2 \times \frac{1}{2}mv^2}{r} = \frac{2E_k}{r} = \frac{2 \times 3.0 \times 10^{10}}{8.0 \times 10^7}$$

= 750 N (ANS)

This is another example of combining the Kinetic Energy formula with the Centripetal Force expression from circular motion

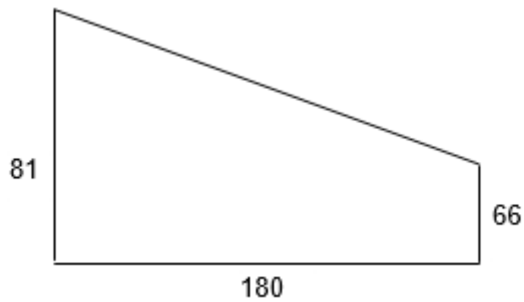
Question 279

A balance will work in any place where there is a gravitational field by comparing the gravitational attraction on two masses, so it will work on the Moon. In a satellite in orbit, the balance and masses would all be in free fall so the gravitational attraction [apparent weight] will be zero.

Question 280

The original kinetic energy will be converted to Gravitational Potential Energy as it moves through the Gravitational field. If the graph supplied was a Force vs height graph, the energy thus transformed would be given by the area under that graph. To convert the g vs height graph, the 'g' values are multiplied by the mass involved – 50 kg.

The area is approximately a Trapezium.



Area $= (81 + 66)/2 \times 180\ 000$

$= 13\ 230\ 000$ J

$\therefore \frac{1}{2} \times 50 \times v^2 = 13\ 230\ 000$

$\therefore$ **$v = 730\ ms^{-1}$ (ANS)**

Question 281

Rearrange a formula involving G such as

$mg = \frac{GMm}{R^2}$ then make G the subject

$$\therefore G = \frac{gR^2}{M} = \frac{m \times m^2}{s^2 \times kg}$$

Substitute the units of the other variables.

$\therefore$ **B (ANS)**

Question 282

See the notes for the derivation of Kepler's Law.

$$\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2}$$

$$\therefore R^3 = \frac{GM}{4\pi^2}T^2$$

This is the standard manipulation of the formula obtained when equating the Gravitational field strength to the centripetal acceleration.

Question 283

$$\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2}$$

$$\therefore M = \frac{4\pi^2 R^3}{GT^2}$$

$$\frac{4 \times \pi^2 \times (3.8 \times 10^8)^3}{6.67 \times 10^{-11} \times (28 \times 24 \times 3600)^2}$$

$\therefore$ **M = 5.5 × 10^{24} kg (ANS)**

Question 284

Use the standard formulae and rearrange to give r = ??

$$\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2}$$

$$\therefore R^3 = \frac{GMT^2}{4\pi^2}$$

$\therefore$ $R^3 = 6.67 \times 10^{-11} \times 6.4 \times 10^{24} \times (24 \times 60 \times 60)^2/(4\pi^2)$

$\therefore$ $R^3 = 8.07 \times 10^{22}$

$\therefore$ $R = 4.3 \times 10^7$ m

To find the altitude, you need to subtract the radius of the Earth,

$\therefore$ $4.3 \times 10^7 - 6.4 \times 10^6$

= **3.66 × 10^7 m (ANS)**

Question 285

$$\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2} \Rightarrow T^2 = \frac{4\pi R^3}{GM}$$

From this equation the period is independent of the mass of the satellite. If they have the same radius of orbit and the same period, then they must both have the same speed.

Question 286 (2010 Q18, 2m, 60%)

$$\text{Weight} = \frac{GM_e m_{iss}}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 3.04 \times 10^5}{(6.72 \times 10^6)^2}$$

= 2.69 × 10^6 N (ANS)

Question 287 (2010 Q19, 2m, 50%)

The period can be calculated from Kepler's Law:

$$\frac{GM}{4\pi^2} = \frac{R^3}{T^2}$$

$$\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4\pi^2} = \frac{(6.72 \times 10^6)^3}{T^2}$$

$$\therefore T^2 = \frac{(6.72 \times 10^6)^3 \times 4\pi^2}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}$$

$\therefore$ **T = 5480 secs (ANS)**

This question can also be done using

$$F = \frac{mv^2}{r}$$

$$= \frac{m}{r}\left(\frac{2\Pi r}{T}\right)^2$$

$$= \frac{m4\Pi^2 r}{T^2}$$

$$\therefore F = \frac{m4\Pi^2 r}{T^2}$$

$$\therefore T^2 = \frac{m4\Pi^2 r}{F}$$

$$\therefore T^2 = \frac{4\pi^2 \times (6.72 \times 10^6)}{2.69 \times 10^6}$$

$\therefore$ **T = 5480 secs (ANS)**

Question 288 (2010 Q20, 1m, 60%)

Same (ANS)

From Kepler's Law, the period is independent of the mass of the satellite.

Question 289 (2011 Q21, 1m, 80%)

W = mg, if the visitor weighs the same then g must equal 10 on both the planet and the Earth.

$\therefore$ **10 N kg^{-1} (ANS)**

Question 290 (2011 Q22, 2m, 50%)

$$\text{Use } F = mg = \frac{GMm}{r^2}$$

$$\therefore g_{Visitor} = \frac{GM_V}{r_V^2}$$

$$\therefore 10 = \frac{GM_V}{r_V^2}$$

$$\therefore 10 = \frac{6.67 \times 10^{-11} \times M_V}{(30 \times 10^3)^2}$$

$\therefore M_V = \dfrac{10 \times (30 \times 10^3)^2}{6.67 \times 10^{-11}}$

$\therefore$ M_V = 1.35 × 10^{20} kg (ANS)

Question 291 (2011 Q23, 2m, 50%)

Use $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$

$\therefore \dfrac{GM}{r} = v^2$

Use $v = \dfrac{2\pi r}{T}$

$\therefore \dfrac{GM}{r} = \dfrac{4\pi^2 r^2}{T^2}$

$\therefore T^2 = \dfrac{4\pi^2 r^3}{GM}$

$\therefore T^2 = \dfrac{4\pi^2 \times (1 \times 10^9)^3}{6.67 \times 10^{-11} \times 5.7 \times 10^{25}}$

$\therefore T^2 = 1.04 \times 10^{13}$

$\therefore$ T = 3.2 × 10^6 secs (ANS)

Question 292 (2012 Q8a, 4m, 58%)

Use $F = \dfrac{GMm}{R^2}$ and $F = \dfrac{mv^2}{R}$, combined with $v = \dfrac{2\Pi R}{T}$ to get $\dfrac{R^3}{T^2} = \dfrac{GM}{4\Pi^2}$.

This is known as Kepler's Law. It is not on the course, but it is extremely useful.

$$R^3 = \frac{GMT^2}{4\Pi^2}$$

$$\therefore R^3 = \frac{6.67 \times 10^{-11} \text{ x } 7.36 \times 10^{22} \times (2 \times 60 \times 60)^2}{4\Pi^2}$$

$\therefore R^3 = 6.45 \times 10^{18}$

$\therefore R = 1.86 \times 10^6$

This is the radius of orbit, the question asks for the height above the moon's surface.

$\therefore h = 1.86 \times 10^6 - 1.74 \times 10^6$

$\therefore$ h = 1.22 × 10^5 m (ANS)

Question 293 (2012 Q8b, 2m, 60%)

Weightlessness is when the weight is zero, i.e. g = 0.

Apparent weightlessness is when the normal reaction is zero. This is when the object is in free fall.

The astronauts in Apollo 11, are apparently weightless, because the satellite is in free fall.

Question 294 (2013 Q7a, 1m, 40%)

Even though it is quite clearly explained, I don't like the language of 'geostationary' as it is not on the course any more. Clearly the examiners think that it is on the course!!!

Period = 24 hours

= 24 × 60 × 60

= 8.64 × 10^4 s (ANS)

Question 295 (2013 Q7b, 2m, 40%)

This is solved quickest using Kepler's Law, which isn't on the course. Clearly the examiners think that it is on the course!!!

Use $\dfrac{r^3}{t^2} = \dfrac{GM}{4\pi^2}$ which is Kepler's Law.

$$\therefore r^3 = \frac{6.7 \times 10^{-11} \times 6.0 \times 10^{24} (8.64 \times 10^4)^2}{4\pi^2}$$

$\therefore r^3 = 7.60 \times 10^{22}$

$\therefore$ r = 4.24 × 10^7 m (ANS)

Question 296 (2013 Q7c, 3m, 40%)

Weight is the force of attraction between her and the Earth. It is given by W = mg, where g is the acceleration due to gravity at that height above the Earth's surface.

Weightlessness exists only where the gravitational field strength is zero. This is not true in this situation.

Apparent weightlessness occurs when force due to gravity is the only force acting, the object is in free fall or the normal reaction force is zero. She is in orbit around the Earth so she will experience apparent weightlessness.

Question 297 (2014 Q5a, 4m, 60%)

Period = 1200 hours

= 1200 × 60 × 60

= 4.32 × 10^6 s

Use $\dfrac{r^3}{t^2} = \dfrac{GM}{4\pi^2}$ which is Kepler's Law.

$$\therefore \frac{(7.0 \times 10^{10})^3}{(4.32 \times 10^6)^2} = \frac{6.67 \times 10^{-11} \times M}{4\pi^2}$$

$$\therefore M = \frac{1.354 \times 10^{34}}{1244.78208}$$

$\therefore M = 1.09 \times 10^{31}$

$\therefore$ M = 1.1 × 10^{31} (ANS)

Question 298 (2014 Q5b, 2m, 40%)

No.

From $\frac{r^3}{t^2} = \frac{GM}{4\pi^2}$, if we only know ‘r’, ‘t’, ‘G’ and ‘M’, we can’t find ‘m’, the mass of the planet.