Making music

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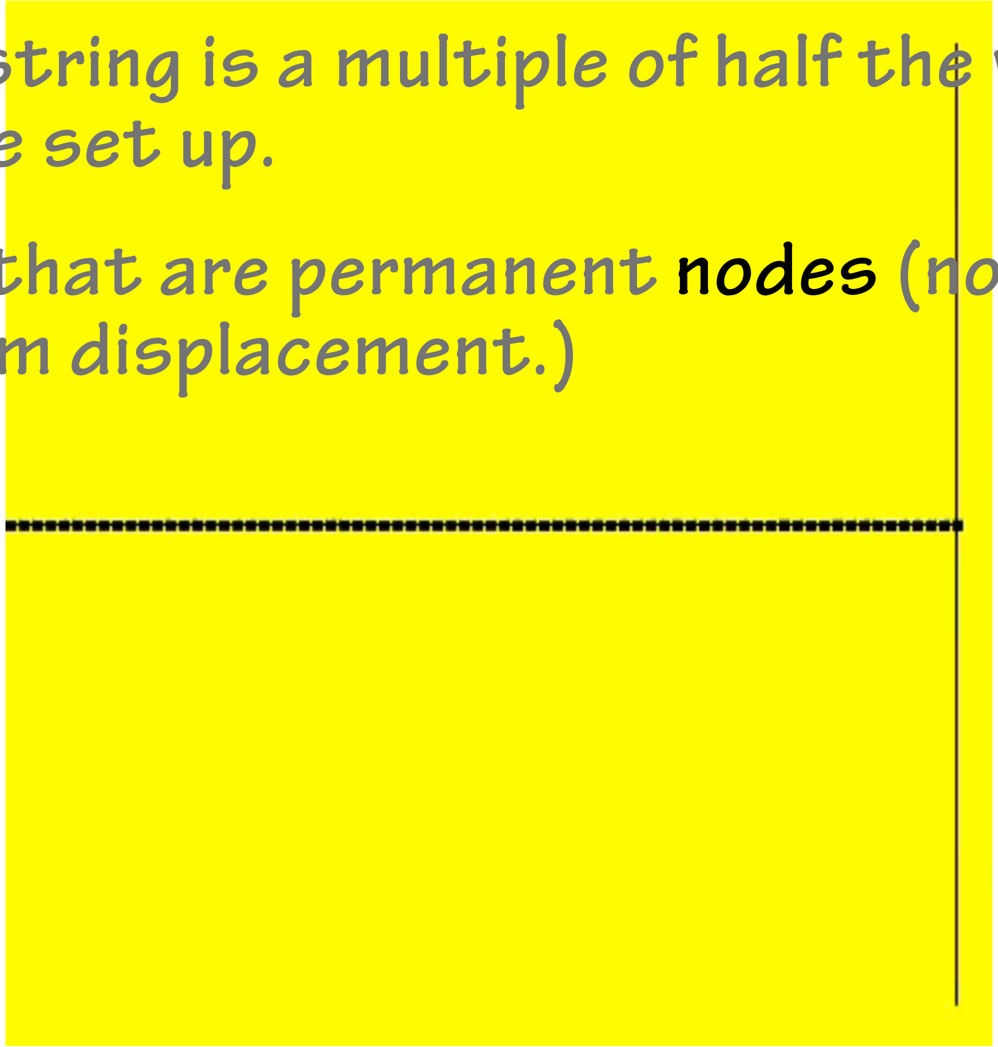
Standing waves in strings

- Waves that travels along a string will be reflected at the end.
- These reflections will cause interference with oncoming waves.
- **Constructive interference**: waves add together.
- **Destructive interference**: waves cancel each other out.
- If the length of the string is a multiple of half the wavelength, a **standing wave** will be set up.
- There will be points that are permanent **nodes** (no displacement) and **anti-nodes** (maximum displacement.)



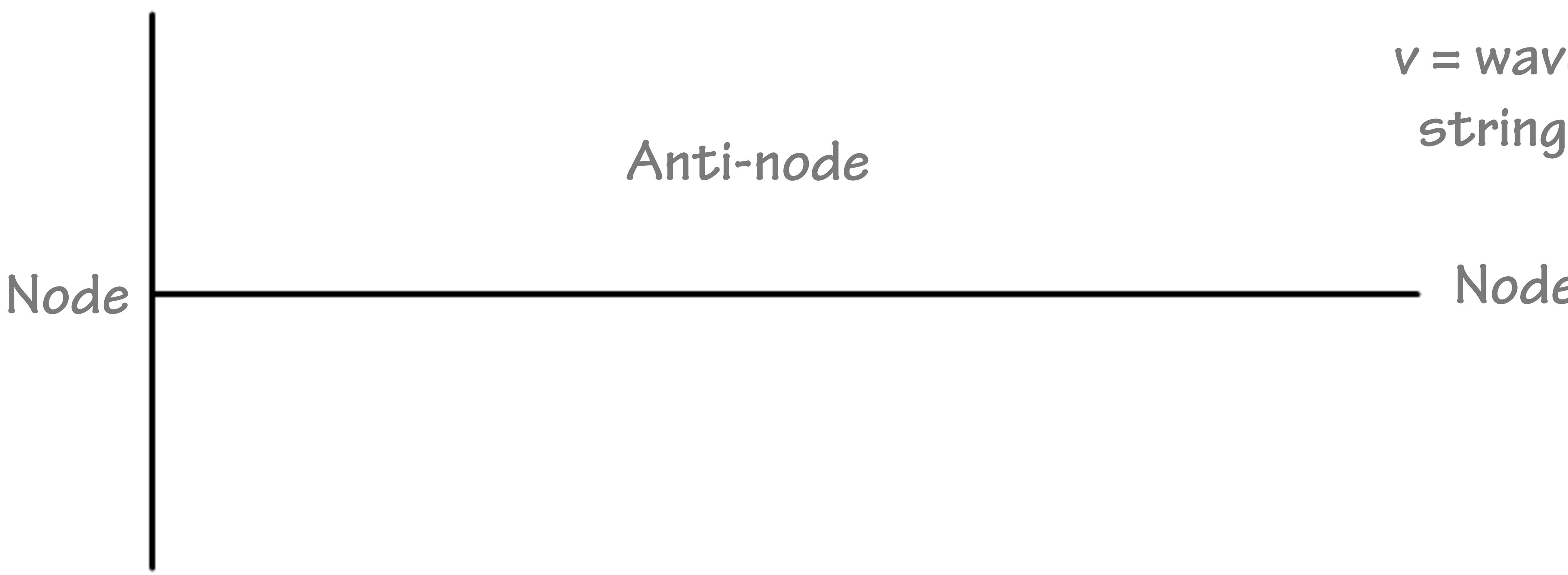
Standing waves in strings

- A guitar string is fixed out both ends.
- These points act as **displacement nodes** - there can be no movement at those points.
- The **fundamental wavelength** is twice the length.
- This is the the **first harmonic**.

$$\lambda_1 = 2L$$

$$f_1 = \frac{v}{2L}$$

v = wave speed in string (not air)



Speed of waves in strings

- Fundamental frequency is a constant for a string at a particular tension.
- The fundamental wavelength is fixed - adjusting the tension on the string will change the wave speed, fundamental frequency & harmonics.
- Wave speed - depends on tension, mass / length. (Compare this to the equation for speed in a solid / gas).
- Waves will travel faster in a tighter string, (or in a lighter string).
- *eg a 5.0 grams /meter, 64 cm metre string under 80N of tension:*

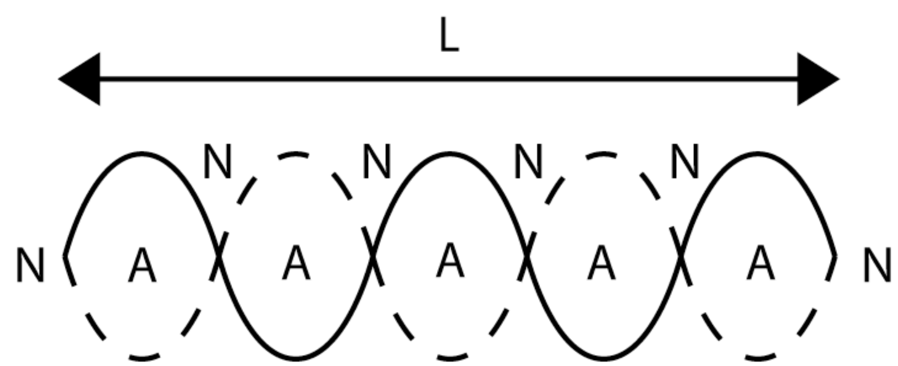
$$v=\sqrt{\frac{F_T}{m/L}} \qquad v=\sqrt{\frac{80N}{0.005kg/m}} \qquad v \approx 1.3 \times 10^2 m/s$$

$$f_1 = \frac{v}{\lambda_1} \qquad f_1 = \frac{130\ m/s}{1.28m} \approx 100Hz$$

Standing waves in strings - other harmonics.

- Other harmonics exist, where the wavelengths are fractions of the fundamental.
- Harmonic frequencies are multiples of the fundamentals.

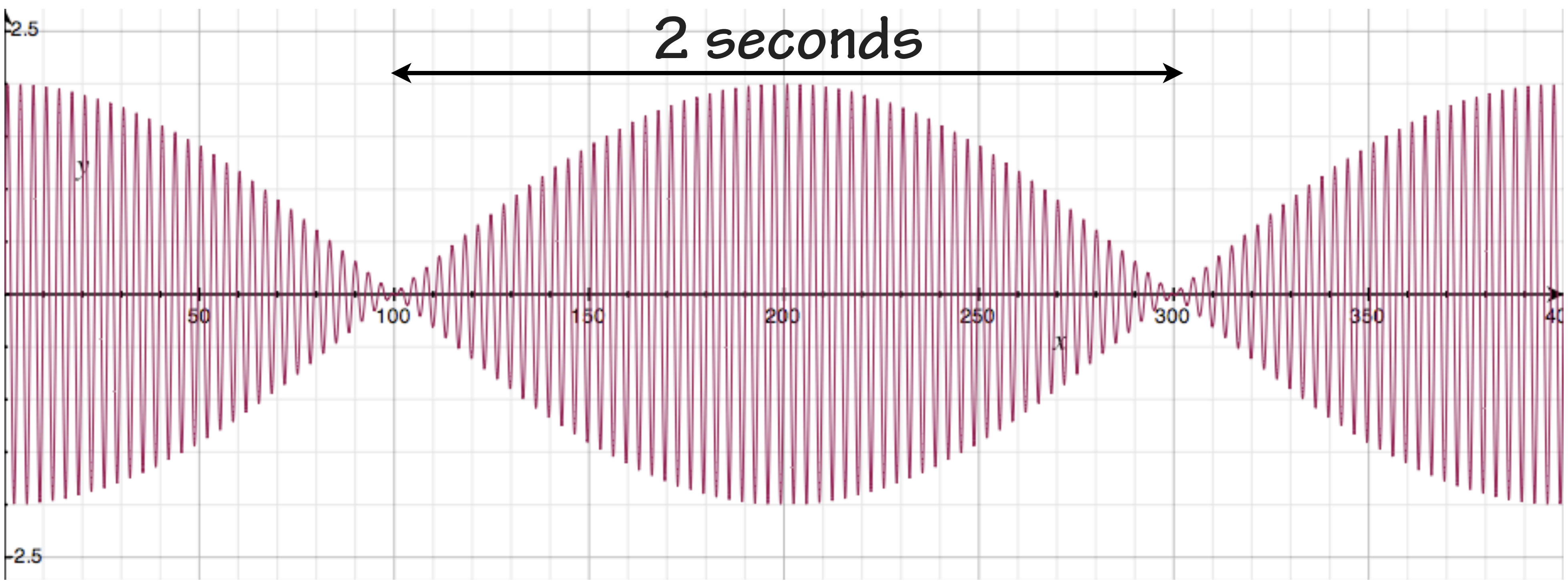
Harmonic	Number of half waves in string	Wavelength	Frequency
1	1	2L	v / 2L
2	2	2L / 2 = L	2v / 2L = v / L
3	3	2L / 3	3v / 2L
4	4	2L / 4 = L / 2	4v / 2L = 2v / L
5	5	2L / 5	5v / 2L



fifth harmonic

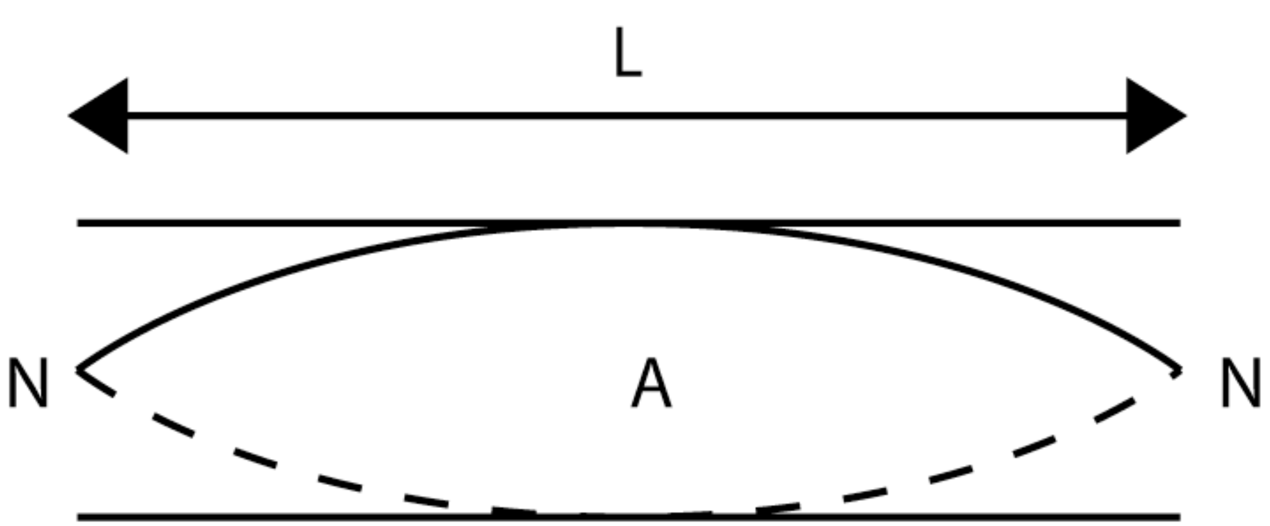
Tuning instruments - Beats

- An out of key string has a different wave speed (V) and therefore a different set of harmonics.
- Strings can be tuned by using a tuning fork; an in tune string will have the same frequency.
- The two sound waves will interfere to cause a combined wave.
- An out of tune string will produce a **beat**.
- Beat frequency = $f_2 - f_1$ (300.5 Hz - 300 Hz = 0.5 Hz)



Open ended pipes

- Blowing across a pipe will produce a series of vibrations; only the harmonics can resonate.
- Each end of the pipe is at atmospheric pressure and must act as a **pressure node**. (These points are also displacement anti-nodes.)
- The first harmonic is one that has a wave twice as long as the pipe.
- eg. for a pipe that is 50cm long, the fundamental frequency is 340Hz.



first harmonic

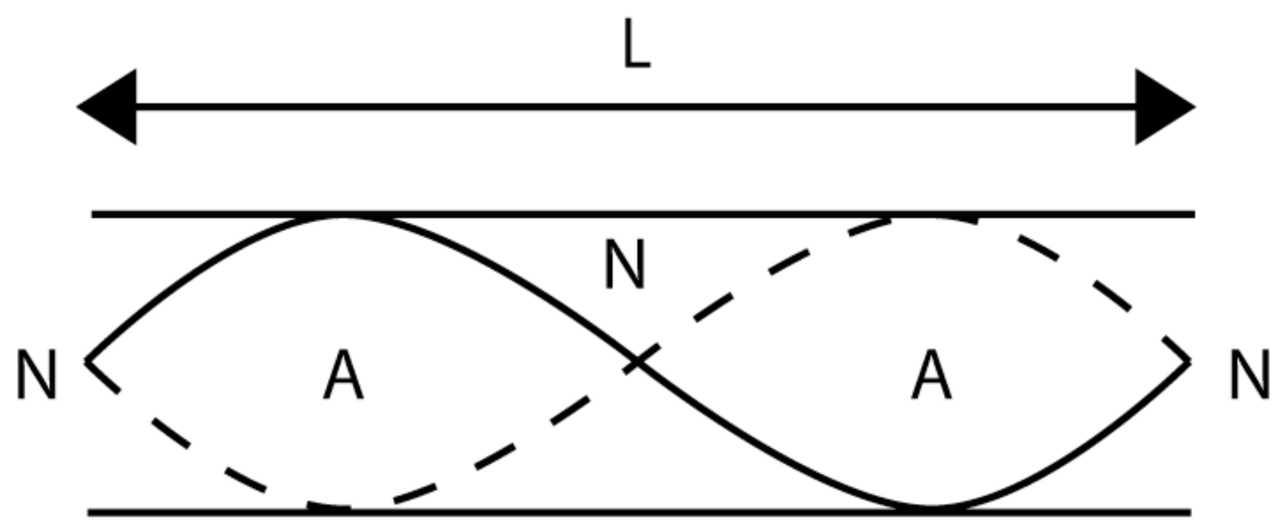
$$\lambda = 2L \qquad \lambda = 2 \times 0.5\ \text{m} = 1\ \text{m}$$

$$f = \frac{v}{2L} \qquad f = \frac{340\ \text{m/s}}{2 \times 0.5\ \text{m}}$$

$$f = 340\ Hz$$

Open ended pipe harmonics

- Frequencies that are multiples of the fundamental can also resonate.



second harmonic

eg. the second harmonic is twice the frequency of the fundamental

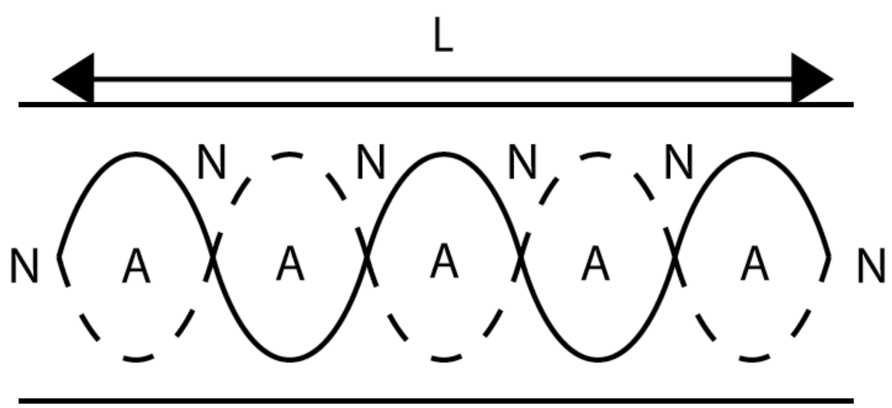
$$\lambda_2 = \frac{2L}{2} = L \qquad \lambda_2 = \frac{2 \times 0.5m}{2} = 0.5m \qquad f_2 = \frac{340\ \text{m/s}}{0.5\ \text{m}} = 680\ \text{Hz}$$

$$f_2 = 2f_1 = 680\ Hz$$

Open ended pipe harmonics

- Other harmonics exist, where the wavelengths are fractions of the fundamental.
- Harmonic frequencies are multiples of the fundamentals.

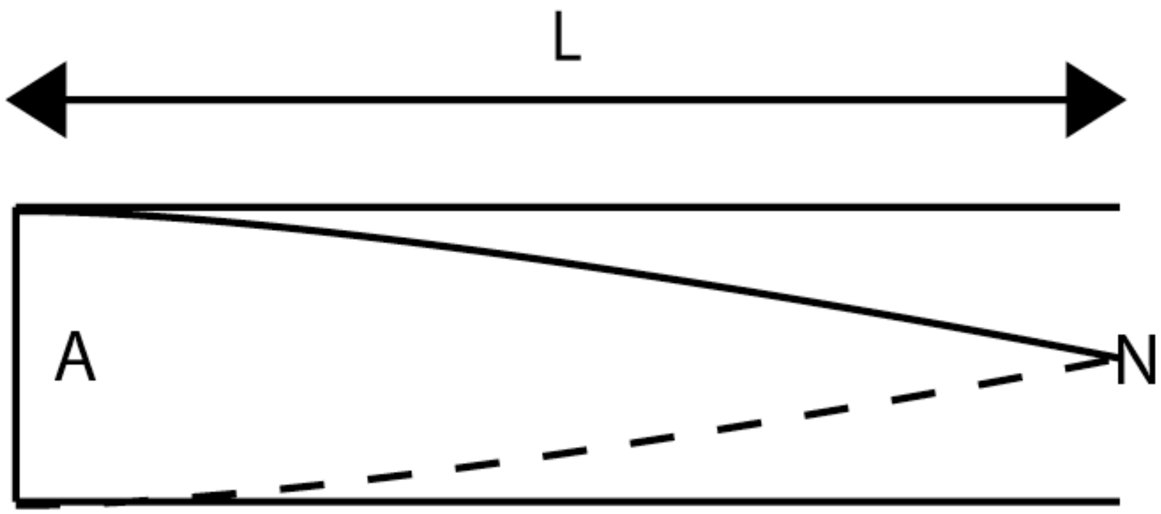
Harmonic	Number of half waves in pipe	Wavelength	Frequency	Example
1	1	2L	v / 2L	340 Hz
2	2	2L / 2 = L	2v / 2L = v / L	680 Hz
3	3	2L / 3	3v / 2L	1020 Hz
4	4	2L / 4 = L / 2	4v / 2L = 2v / L	1360 Hz
5	5	2L / 5	5v / 2L	1700 Hz



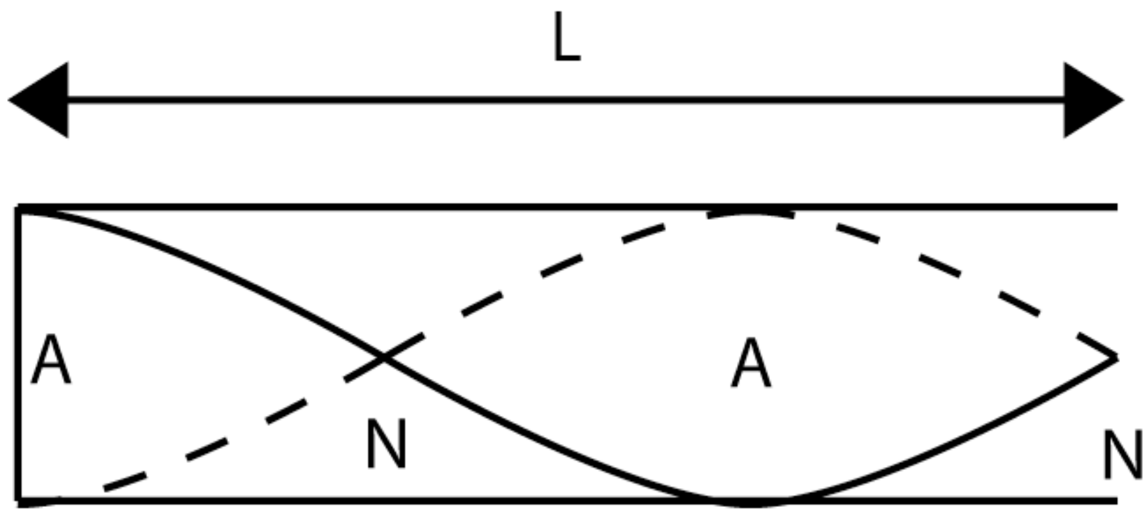
fifth harmonic

Closed pipes

- A closed pipe (only at one end!) has a pressure node at the open end and an anti-node at the closed end.
- The first harmonic wave is four times the length of the pipe.
- Only the odd numbered harmonics can exist; n=1,3,5. The next resonant frequency above the fundamental will be the third harmonic.
- eg. A closed pipe with a length of 0.5m will only allow the frequencies 170 Hz, 510 Hz, 850 Hz etc to resonate.



first harmonic



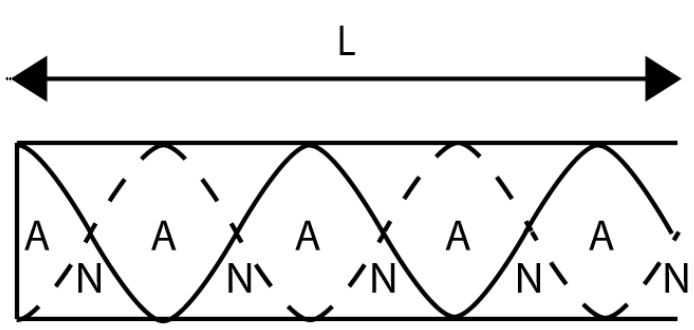
third harmonic

$$\lambda = 4L \qquad f = \frac{v}{4L}$$

Closed pipes harmonics

- Other harmonics exist, where the wavelengths are fractions of the fundamental.
- Harmonic frequencies are odd multiples of the fundamentals.

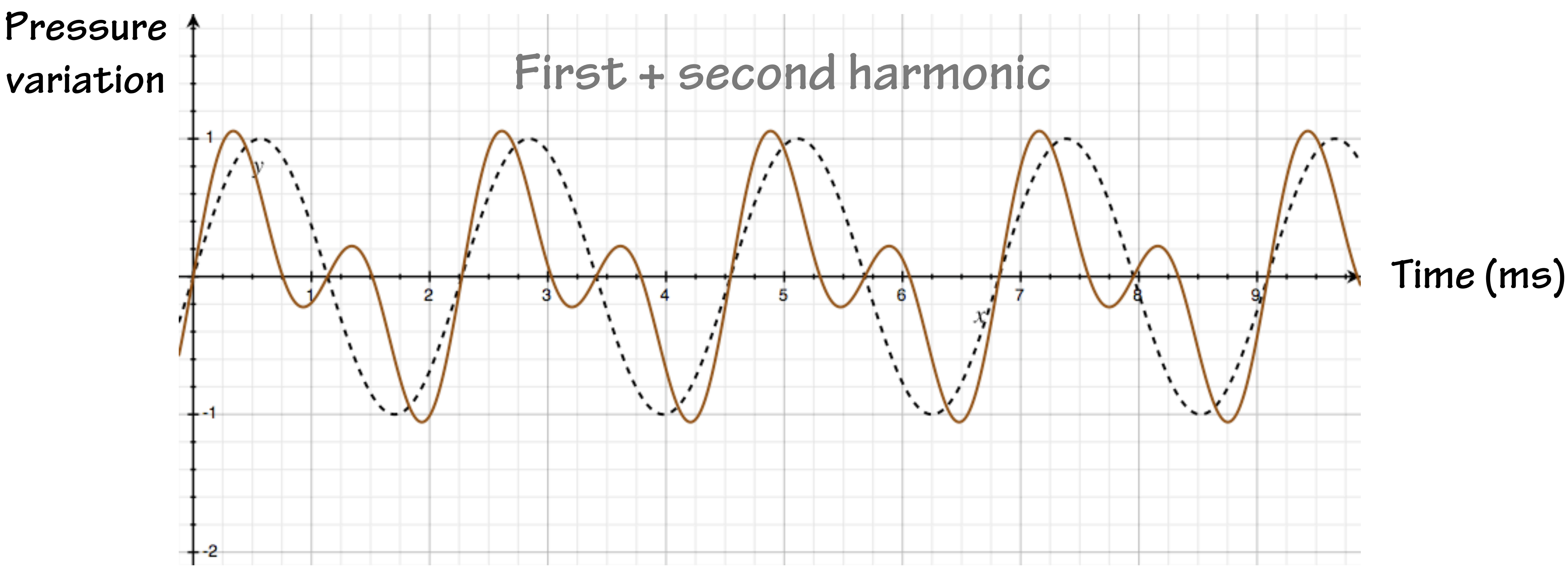
Harmonic	Number of quarter waves in pipe	Wavelength	Frequency	Example
1	1	4L	v / 4L	170 Hz
3	3	4L / 3	3v / 4L	510 Hz
5	5	4L / 5	5v / 4L	850 Hz
7	7	4L / 7	7v / 4L	1190 Hz
9	9	4L / 9	9v / 4L	1530 Hz



ninth harmonic

Harmonics & timbre

- Different instruments will have a different sound even though they are playing the same note.
- Other harmonics will be also be resonating - these will be a characteristic of the material & shape of the instrument.
- The waveform will be combination of all of the harmonics (intensity decreases with frequency).
- This is responsible for the **tone** or **timbre** of the sound.



Spectral analysis

- Piano note, fundamental frequency ~ 500 Hz.
- Other multiples of this are also present (1000 Hz, 1500 Hz, 2000 Hz...)
- Other strings are forced to resonate if they share some harmonics.